On the Lattice Isomorphism Problem, Cryptography and the Signature Scheme HAWK

Léo Ducas, Eamonn Postlethwaite, Ludo Pulles (CWI, Cryptology Group), Wessel van Woerden (Université de Bordeaux, IMB).

Centrum Wiskunde & Informatica

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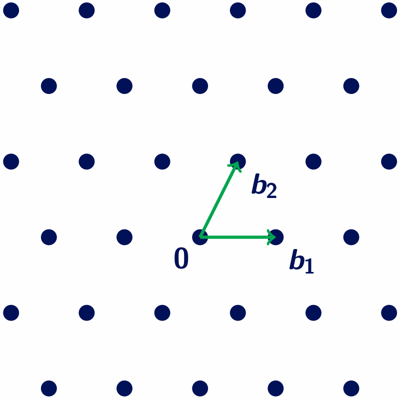
- Better lattice $\Longrightarrow$ better efficiency and security.
- HAWK: a simple and efficient signature scheme from $\mathbb{Z}^n$.

Lattices

Lattice $\mathcal{L}(\boldsymbol{B}) := \{\sum_i x_i \boldsymbol{b}_i : \boldsymbol{x} \in \mathbb{Z}^n\} \subset \mathbb{R}^n$



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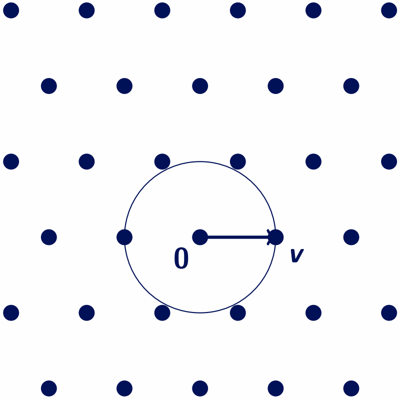

First minimum

$$\lambda_1(\mathcal{L}) := \min_{x \in \mathcal{L} \setminus \{0\}} \|x\|_2$$

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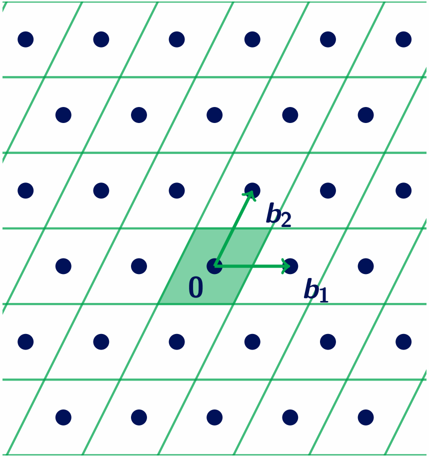

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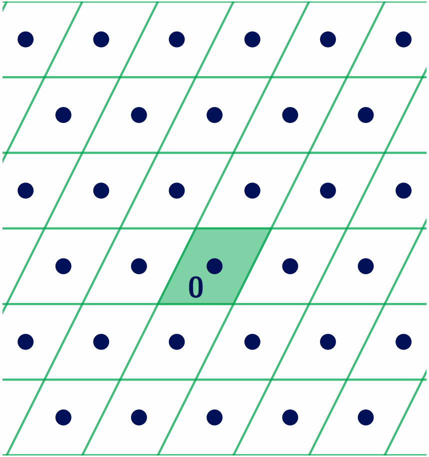

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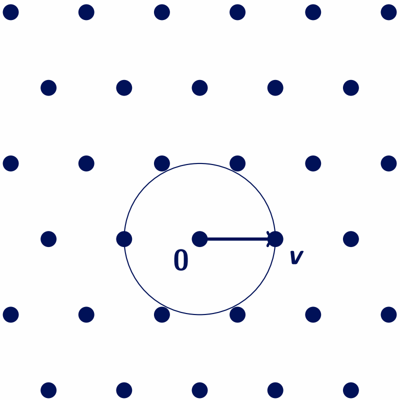
Minkowski's Theorem

$$\lambda_1(\mathcal{L}) \leq \underbrace{2\frac{\det(\mathcal{L})^{1/n}}{\mathrm{vol}(\mathcal{B}^n)^{1/n}}}_{\mathrm{Mk}(\mathcal{L})} \leq \sqrt{n}\det(\mathcal{L})^{1/n}$$

Lattice $\mathcal{L} \subset \mathbb{R}^n$



SVP

Find a *shortest* nonzero vector $\boldsymbol{v} \in \mathcal{L}$ of length $\lambda_1(\mathcal{L}) \leq \mathsf{Mk}(\mathcal{L})$.

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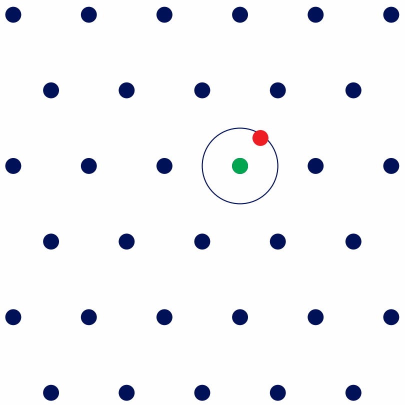

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BDD

Given a target $t = v + e \in \mathbb{R}^n$ with

$$v \in \mathcal{L} \text{ and } \|e\| < \rho \leq \frac{1}{2}\lambda_1(\mathcal{L}) \leq \frac{1}{2}\mathsf{Mk}(\mathcal{L}),$$

recover $v \in \mathcal{L}$.

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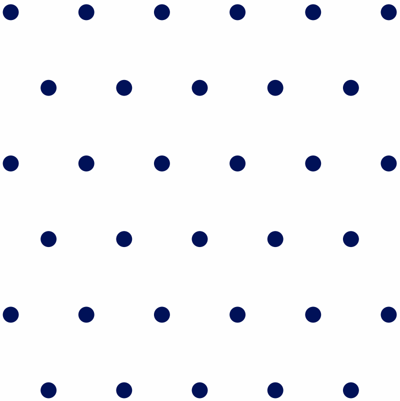

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Hardness depends on the *gaps*!

Hard Problems

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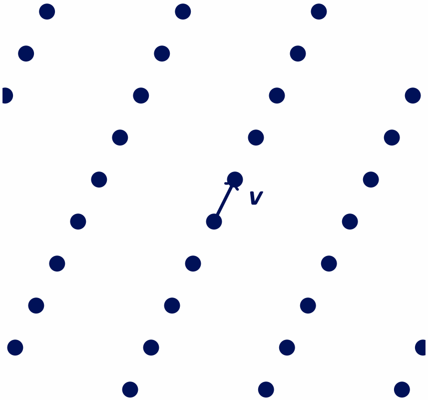

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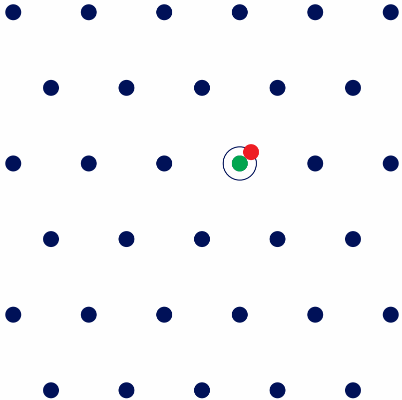

<u>SVP</u>

Find a *shortest* <u>nonzero</u> vector $\boldsymbol{v} \in \mathcal{L}$ of length $\underbrace{\lambda_1(\mathcal{L}) \leq \mathsf{Mk}(\mathcal{L})}_{\mathsf{gap}(\mathcal{L})}$.

<u>BDD</u>

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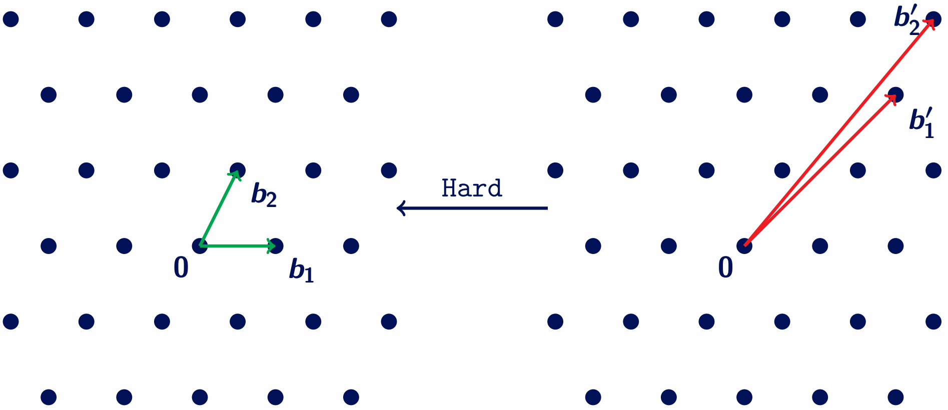
Good basis (Secret key)   Bad basis (Public key)



$\boldsymbol{B}$ $\boldsymbol{B}' = \boldsymbol{B} \cdot \boldsymbol{U}, \;\; \boldsymbol{U} \in \mathrm{GL}_d(\mathbb{Z})$

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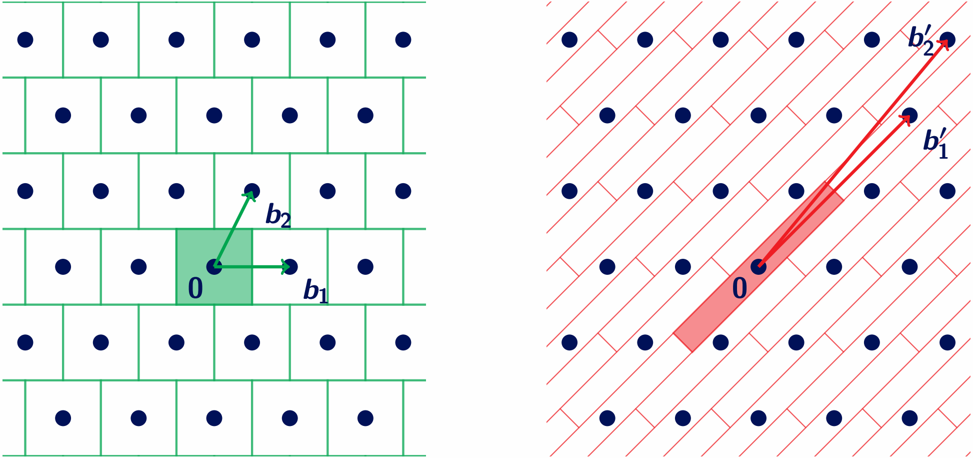

Babai's nearest plane algorithm

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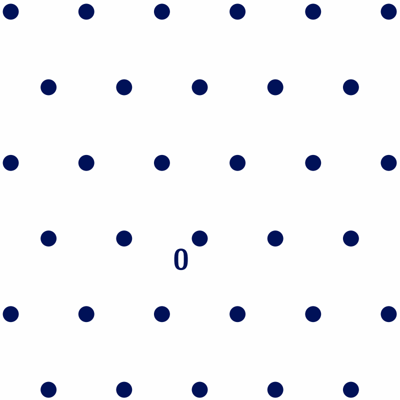

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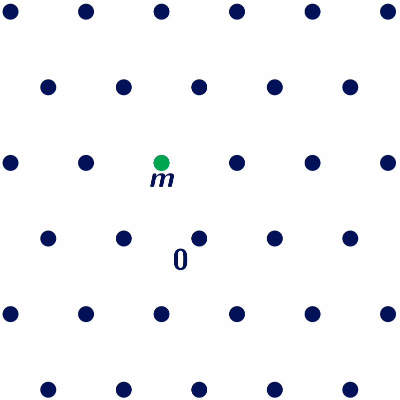

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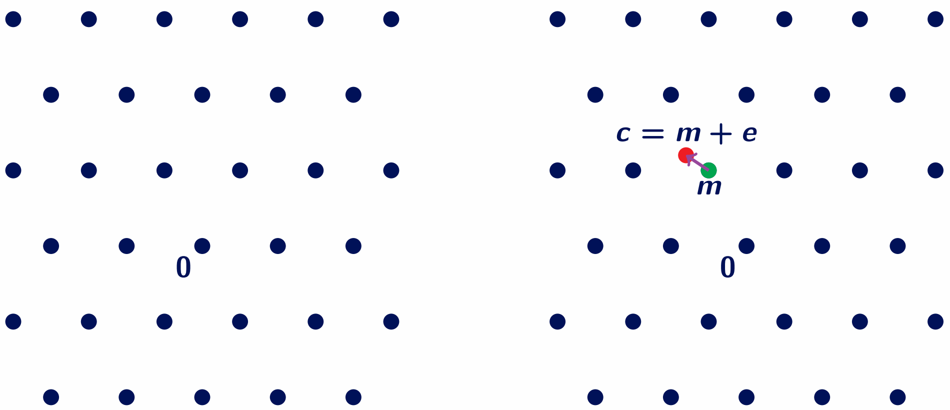

Encrypt by adding a small error

Encryption, legacy approach

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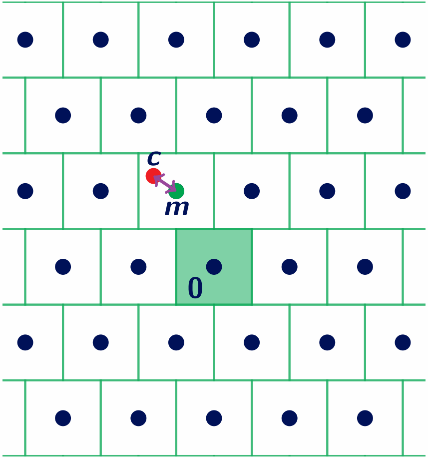

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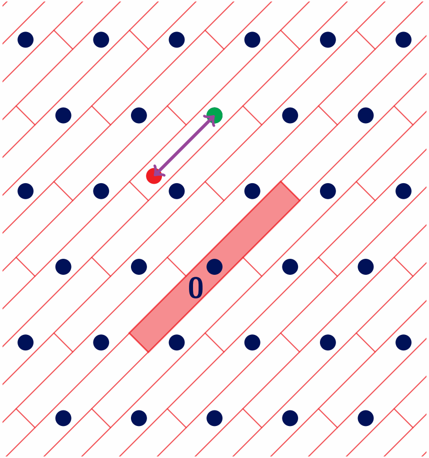

Decrypt using the good basis

Large gap

Current lattice based crypto relies on hardness of decoding with

$$\mathrm{gap}(\mathcal{L}, \rho) \geq \Omega(\sqrt{n}).$$

Broken by SVP in dimension $\beta \leq n/2 + o(n)$, e.g. $n = 1024 \implies \beta \approx 450$.

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An example: Prime Lattice [CR88]

Let $p_1, \ldots, p_n$ be distinct small primes not dividing m, we define:

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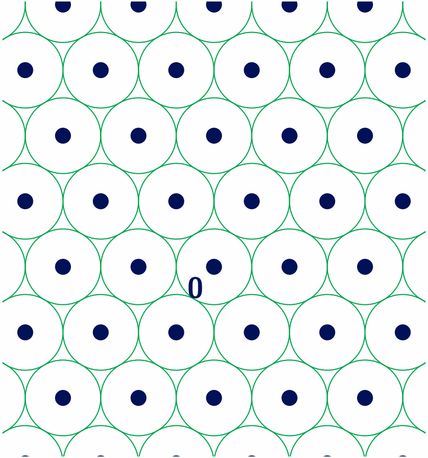
- Efficiently decode up to large radius ρ by trial division.
- With the right parameters $\mathsf{gap}(\mathcal{L}_{\texttt{prime}}, \rho) = \Theta(\log(n))$ [DP19].

How to hide the remarkable lattice?

Good lattice (~~Secret key~~)



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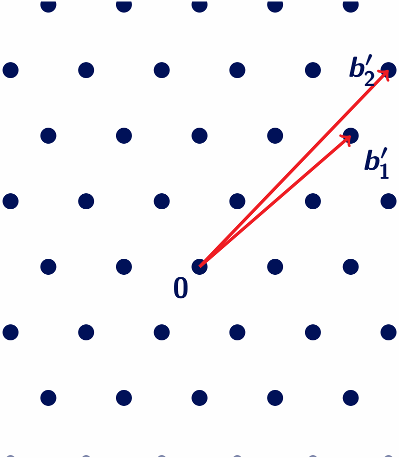

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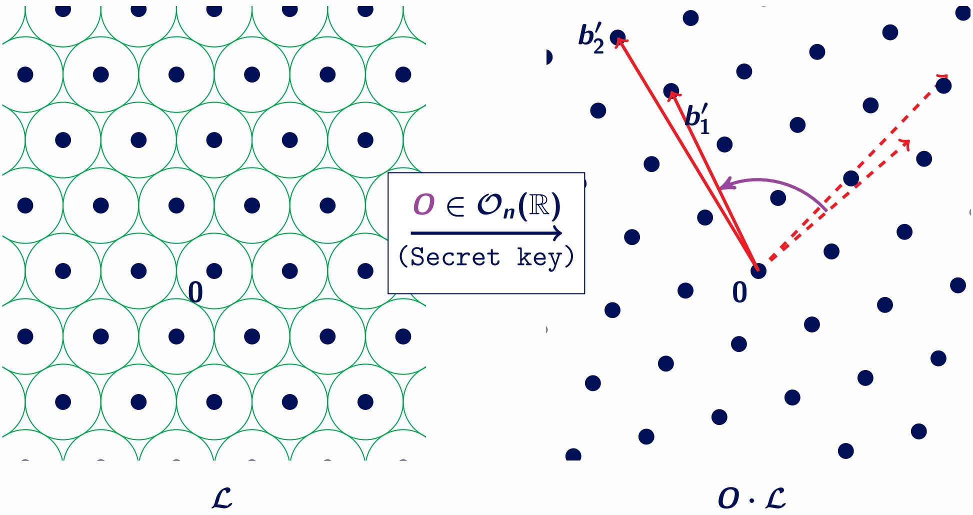

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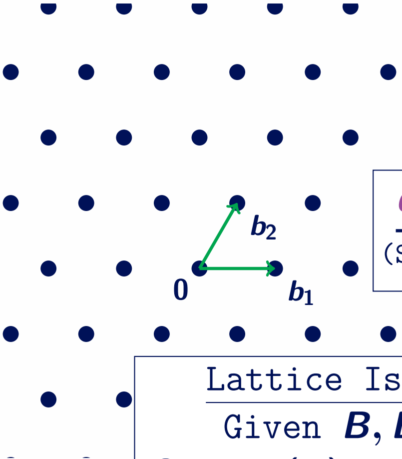

$\xrightarrow[\text{(Secret key)}]{O \in \mathcal{O}_n(\mathbb{R})}$

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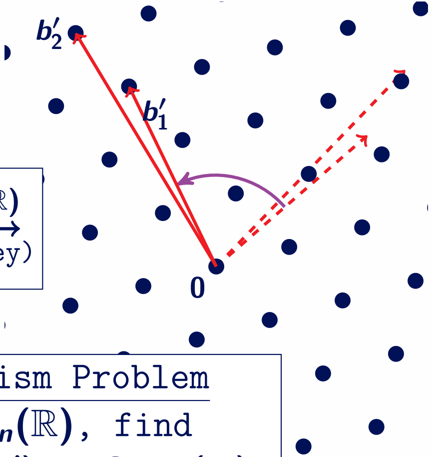

Lattice Isomorphism Problem

Given $B, B' \in \mathrm{GL}_n(\mathbb{R})$, find $O \in \mathcal{O}_n(\mathbb{R})$ s.t. $\mathcal{L}(B') = O \cdot \mathcal{L}(B)$.

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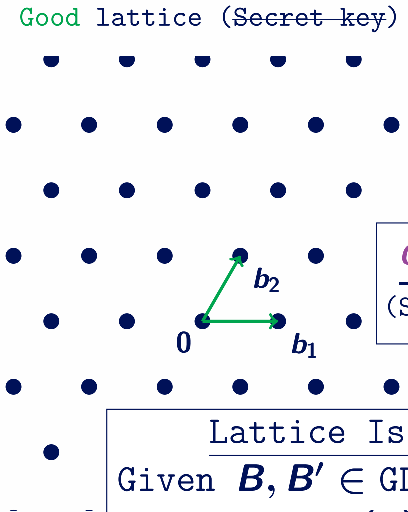

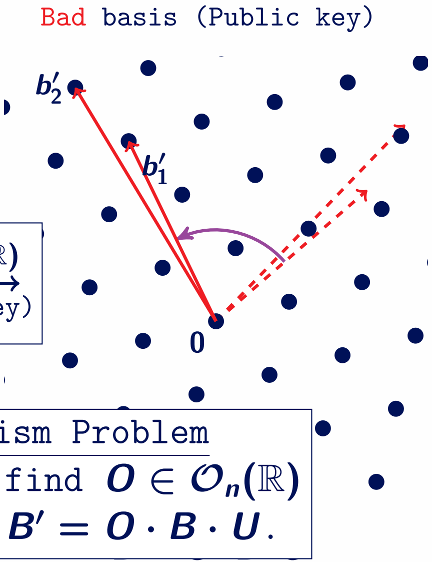

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Lattice Isomorphism Problem

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- The lattice analogue of 'vintage' McEliece $\mathbf{G}' = \mathbf{P} \cdot \mathbf{G} \cdot \mathbf{S}$,
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- The lattice analogue of 'vintage' McEliece $G' = P \cdot G \cdot S$,
- and Oil and Vinegar $\mathcal{P} = \mathcal{Q} \circ \mathcal{S}$.
- Best known attacks require to solve SVP.

Algorithms

- $\mathrm{Min}(\mathcal{L}(B')) = O \cdot \mathrm{Min}(\mathcal{L}(B))$.
- Best practical algorithm: backtrack search all isometries between the sets of short vectors.
- Best proven algorithm uses short primal and dual vectors ($n^{O(n)}$ time and space).

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Sidestep real values!

$\boldsymbol{O} \in \mathcal{O}_n(\mathbb{R})$

Sample $\boldsymbol{U} \in \mathrm{GL}_{\boldsymbol{n}}(\mathbb{Z})$ s.t.
$\boldsymbol{B}'$ is independent of $\boldsymbol{B}$.

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$$Q := B^t B \in \mathcal{S}_n^{>0}$$

Lattices $\implies$ Quadratic Forms

$$(\mathcal{L} \subset \mathbb{R}^n, \langle x, y \rangle) \implies (\mathbb{Z}^n, \langle x, y \rangle_Q := x^t Q y)$$

Keep the geometry, forget the embedding.

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LIP restated:

Find $U \in \mathrm{GL}_n(\mathbb{Z})$ s.t. $Q' = U^t Q U$.

An average-case distribution

Unimodular $\boldsymbol{U} \in \mathrm{GL}_{\boldsymbol{n}}(\mathbb{Z})$

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+ Efficient sampler $(Q', U) \leftarrow \mathrm{Sample}_\sigma(Q)$ s.t. $Q' \sim \mathcal{D}_\sigma([Q])$ and $Q' = U^t Q U$.

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$\boldsymbol{Q}'$ *only depends on the class* $[\boldsymbol{Q}]$ *and not on* $\boldsymbol{Q}$ *itself.*

$\Longrightarrow$ average-case LIP, ZKPoK, identification scheme.

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$\implies$ average-case LIP, ZKPoK, identification scheme.

$\implies$ Worst-case to average-case reduction over $[\boldsymbol{Q}]$.

An average-case distribution

- ac-LIP$^{\boldsymbol{Q}}_{\sigma}$: given $\boldsymbol{Q}$ and $\boldsymbol{Q}' \leftarrow \mathcal{D}_\sigma([\boldsymbol{Q}])$, recover $\boldsymbol{U} \in \mathrm{GL}_n(\mathbb{Z})$.

An average-case distribution

- ac-LIP$_\sigma^Q$: given Q and $Q' \leftarrow \mathcal{D}_\sigma([Q])$, recover $U \in \mathrm{GL}_n(\mathbb{Z})$.
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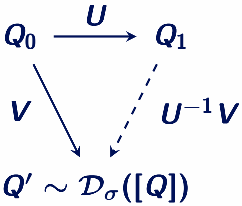

$(Q', V) \leftarrow \mathrm{Sample}_\sigma(Q_0)$

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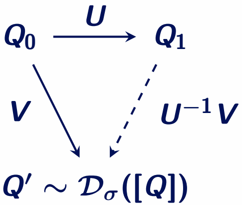
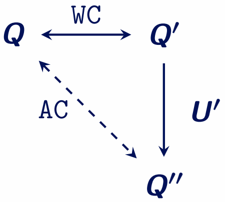


$$(Q', V) \leftarrow \mathrm{Sample}_\sigma(Q_0)$$

- Worst-case to average-case reduction:



$$(Q'', U') \leftarrow \mathrm{Sample}_\sigma(Q')$$

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Best attack: generic lattice reduction.

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Dual SVP attack: $\mathbf{gap}(\mathcal{L}^*)$.

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Decoding attack (BDD): $\mathbf{gap}(\mathcal{L}, \rho)$.

Decodable Lattices

	Primal	Dual	Decoding
Decodable Lattice	$\mathrm{gap}(\mathcal{L})$	$\mathrm{gap}(\mathcal{L}^*)$	$\mathrm{gap}(\mathcal{L}, \rho)$
Random Lattice	$\Theta(1)$	$\Theta(1)$	$2^{\Theta(n)}$
$\mathbb{Z}^n$	$\Theta(\sqrt{n})$	$\Theta(\sqrt{n})$	$\Theta(\sqrt{n})$
NTRU [HPS98]	$\Omega(\alpha)$	$\Omega(\alpha)$	$\Omega(n/\alpha)$
LWE [Ajt99, AP11, MP12]	$\Omega(1)$	$\Omega(\alpha)$	$\Omega(n/\alpha)$
Prime Lattice [CR88, DP19]	$\Theta(\log n)$	$\Omega(\sqrt{n})$	$\Theta(\log n)$
Barnes-Sloane [MP21]	$\Theta(\sqrt{\log n})$	$\Omega(\sqrt{n})$	$\Theta(\sqrt{\log n})$
Reed-Solomon [BP22]	$\Theta(\sqrt{\log n})$	$\Omega(\sqrt{n})$	$\Theta(\sqrt{\log n})$
Barnes-Wall [MN08]	$\Theta(\sqrt[4]{n})$	$\Theta(\sqrt[4]{n})$	$\Theta(\sqrt[4]{n})$

Interesting cases

Decodable lattice $\mathcal{L}$ $\implies$ Encryption scheme.

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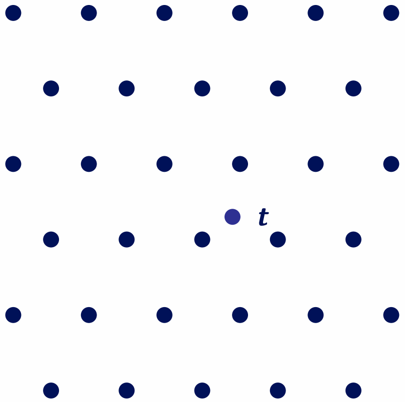
?: gaps $\leq$ poly-log(n),
$$\beta \approx n.$$

FALCON (and MITAKA) use the hash-and-sign design with NTRU lattices.



Sign(m):

- Hash m to a target t.
- (Gaussian) sample a nearby lattice point s using a good basis.

Verify(m, s):

- Hash m to a target t.
- Check $s \in \mathcal{L}$ and $\|s - t\|$ small.

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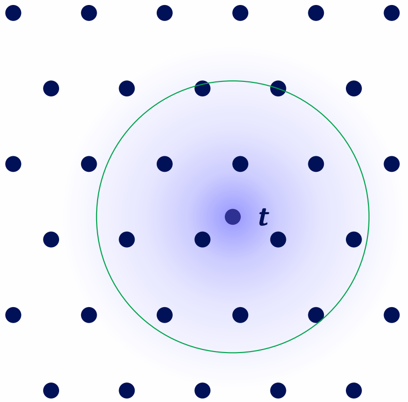

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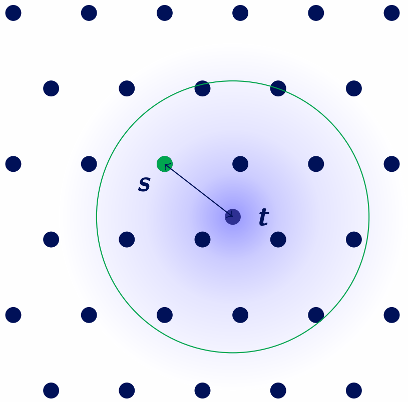

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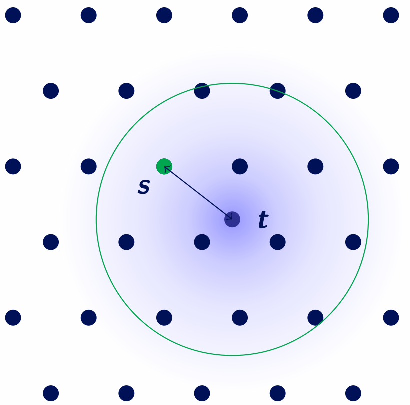

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Complicated and slow!

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How to make it competitive?

1. We add structure: module-LIP.
2. We compress keys and signatures.
3. Only hash to targets in $\frac{1}{2}\mathbb{Z}^n$.

Performance of Hawk

- Hawk has an *isochronous* implementation in C.

	Falcon-512	Hawk-512		Falcon-1024	Hawk-1024	
KeyGen *	**7.95** ms	**4.25** ms	$\downarrow$ /**1.9**	**23.60** ms	**17.88** ms	$\downarrow$ /**1.3**
Sign *	**193** µs	**50** µs	$\downarrow$ /**3.9**	**382** µs	**99** µs	$\downarrow$ /**3.9**
Verify *	**50** µs	**19** µs	$\downarrow$ /**2.6**	**99** µs	**46** µs	$\downarrow$ /**2.2**
$\lvert$**sk**$\rvert$	**1281**	**1153**	$\downarrow$ /**1.1**	**2305**	**2561**	$\uparrow$ $\times$**1.1**
$\lvert$**pk**$\rvert$	**897**	**1006** $\pm$ **6**	$\uparrow$ $\times$**1.2**	**1793**	**2329** $\pm$ **11**	$\uparrow$ $\times$**1.29**
$\lvert$**sig**$\rvert$	**652** $\pm$ **3**	**542** $\pm$ **4**	$\downarrow$ /**1.20**	**1261** $\pm$ **4**	**1195** $\pm$ **6**	$\downarrow$ /**1.06**

Table: Performance on an i5-4590 @**3.30**GHz CPU.
*: AVX2 implementation using floats.

Performance of Hawk

- HAWK has an *isochronous* implementation in C.
- HAWK remains fast when floating points are unavailable.

	FALCON-512	HAWK-512		FALCON-1024	HAWK-1024	
KeyGen *	**7.95** ms	**4.25** ms	↓ /1.9	**23.60** ms	**17.88** ms	↓ /1.3
KeyGen	**19.32** ms	**13.14** ms	↓ /1.5	**54.65** ms	**41.39** ms	↓ /1.3
Sign *	**193** µs	**50** µs	↓ /3.9	**382** µs	**99** µs	↓ /3.9
Sign	**2449** µs	**168**µs	↓ /15	**5273** µs	**343**µs	↓ /15
Verify *	**50** µs	**19** µs	↓ /2.6	**99** µs	**46** µs	↓ /2.2
Verify	**53** µs	**178** µs	↑ ×3.4	**105** µs	**392** µs	↑ ×3.7
$\|sk\|$	**1281**	**1153**	↓ /1.1	**2305**	**2561**	↑ ×1.1
$\|pk\|$	**897**	**1006** ± **6**	↑ ×1.2	**1793**	**2329** ± **11**	↑ ×1.29
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Any lattice $\Longrightarrow$ Identification scheme.

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Thanks! :)